Eur. Phys. J. D **12**, 211–217 (2000)

THE EUROPEAN PHYSICAL JOURNAL D
EDP Sciences

Fermion gases in magnetic fields: a semiclassical treatment

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Received 10 March 2000

Abstract. The study of quantum degenerate gases has received much interest in these last years essentially thanks to the extremely important experimental results of the achievement of Bose-Einstein condensation of atoms and, very recently, of almost complete degeneracy of atomic fermion gases. Here we want to present the results of a semi-analytical method for the study of an interacting degenerate fermion gas based on semiclassical kinetic theory; special care has been devoted to the study of a rotating electron gas, in a cylindrically symmetrical configuration, radially confined by a uniform magnetic field. The model will lead to a particular Thomas-Fermi equation which is generalized to take into account finite temperature and average velocity of the gas, and which is further developed to consider the effects of external fields.

PACS. 05.30.Fk Fermion systems and electron gas – 31.15.Bs Statistical model calculations (including Thomas-Fermi and Thomas-Fermi-Dirac models) – 03.65.Sq Semiclassical theories and applications

1 Introduction

The study of quantum degenerate gases has received much interest in these last years essentially thanks to the extremely important experimental results of the achievement of Bose-Einstein condensation of atoms and, very recently, of almost complete degeneracy of atomic fermion gases [1]. This hopefully will open the path in the future for the realization of a Bose-Einstein condensate of paired fermion atoms.

Ideal trapped atomic fermion gases have been treated theoretically by various authors [2,3], however the difficult investigation of the non-ideal fermion gas has not yet been fully accomplished.

Here we want to present the results of a method [4] for the study of an interacting degenerate electron gas based on semiclassical kinetic theory [5]; in particular we consider a rotating electron gas, in a cylindrically symmetrical configuration, radially trapped (confined) by a uniform magnetic field. This will lead to a particular Thomas-Fermi equation which is generalized to take into account finite temperature and average velocity of the gas, and which is further developed to consider the effects of external fields.

In this approach the exchange and correlation effects have not been taken into account, but it shouldn't be too difficult to include them since the basic Thomas-Fermi model has been extended in this direction in the past.

Several results of the semiclassical kinetic theory coincide indeed with those obtained by a full quantum treatment of the same problem; this really stands as a validation of the bases of our method and approach, which proved to be successful even for the study of certain transport properties of weakly degenerate plasmas and for the calculation of the transport coefficients of electrons in metals at low temperatures, topics which we already considered elsewhere [6]. Kinetic theory in fact gives immediately the possibility of treating directly non-equilibrium cases and of extending equilibrium ones outside equilibrium itself.

The degree of quantum degeneracy of a gas of particles of mass m, temperature T, and mean density n can be estimated through the Sommerfeld's parameter Δ [7]:

$$\Delta = \frac{nh^3}{G(2\pi mkT)^{3/2}} \quad (1)$$

where k is the Boltzmann constant and G an internal factor of degeneracy. The higher is Δ, the higher is the degree of degeneracy of the gas. A gas is said to be completely degenerate when $\Delta \to \infty$, strongly degenerate when $\Delta \gg 1$, weakly degenerate when $\Delta \approx 1$ and non-degenerate when $\Delta \ll 1$ respectively.

The basic semiclassical kinetic equation for the description of degenerate gases is the Boltzmann-Uehling-Uhlenbeck (BUU) equation [8]:

$$\frac{\mathrm{D}f}{\mathrm{D}t} = \int [f'f_1'(1+\xi f)(1+\xi f_1) - ff_1(1+\xi f')(1+\xi f_1')]g\sigma \mathrm{d}\mathbf{\Omega}\mathrm{d}\mathbf{v}_1 \quad (2)$$

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where

$$\frac{\mathrm{D}f}{\mathrm{D}t} = \frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m}\cdot\frac{\partial f}{\partial \mathbf{v}}$$

is the total derivative of the particle distribution function f in the phase space and $\xi = \delta h^3/Gm^3$ with

$$\delta = \begin{cases} 0 & \text{for non-degenerate gas particles,} \\ -1 & \text{for degenerate fermion gas particles,} \\ 1 & \text{for degenerate boson gas particles.} \end{cases}$$

The right-hand-side of the BUU equation is the particle collision term $(\partial f/\partial t)_{\mathrm{Coll.}}$; by definition it must vanish at thermodynamical equilibrium; there exist only two distribution functions that render identically zero the collision term and therefore are representative of equilibrium states: the Fermi function for fermions ($\delta = -1$) and the Bose function for bosons ($\delta = 1$) respectively; these functions can be obtained by solving for the distribution function f the equation $(\partial f/\partial t)_{\mathrm{Coll.}} = 0$ [8]. This means that at equilibrium we are left only with the left-hand-side of the BUU equation equal to zero.

2 The model and its theoretical background [4]

We have seen in the preceding paragraph that the appropriate kinetic equation to describe quantum degenerate systems in the semiclassical approach is the BUU equation. For the case of an electron gas ($G = 2$) it can be written as

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m}\cdot\frac{\partial f}{\partial \mathbf{v}} = \int (f' f_1'(1-\gamma f)(1-\gamma f_1) - f f_1(1-\gamma f')(1-\gamma f_1'))g\sigma(g,\chi)\mathrm{d}\mathbf{\Omega}\mathrm{d}\mathbf{v}_1 \quad (3)$$

where $\gamma = h^3/2m^3$.

Since we are going to study here certain equilibrium properties of the gas it is more convenient to make use of the H theorem and consequently write the equation in the form adequate for the logarithm of the distribution function; if we moreover want to take explicitly into account a finite average velocity $\mathbf{v}_0$ we have finally:

$$(\mathbf{v}_0 + \mathbf{c})\cdot\frac{\partial \ln f}{\partial \mathbf{r}} + \left(\frac{\mathbf{F}}{m} - \left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0\right)\cdot\frac{\partial \ln f}{\partial \mathbf{c}} - \frac{\partial \ln f}{\partial \mathbf{c}}\mathbf{c} : \frac{\partial\,\mathbf{v}_0}{\partial \mathbf{r}} = 0 \quad (4)$$

where obviously $\mathbf{c}$ is the peculiar velocity.

Introducing, as a logical consequence of the state of thermodynamical equilibrium we want to examine, the Fermi distribution function

$$f_{\mathrm{F}}(\mathbf{r},\,\mathbf{c}) = \frac{\gamma^{-1}}{A(\mathbf{r})\mathrm{e}^{\frac{mc^2}{2kT}} + 1} = \frac{\gamma^{-1}}{\mathrm{e}^{\frac{\frac{1}{2}mc^2 - \mu(\mathbf{r})}{kT}} + 1} \quad (5)$$

into equation (4) we get the following equation which must be identically satisfied:

$$\frac{\mathrm{e}^{\frac{mc^2}{2kT}}}{\mathrm{e}^{\frac{mc^2}{2kT}} + 1}\left[(\mathbf{v}_0 + \mathbf{c})\cdot\left(\frac{\partial A}{\partial \mathbf{r}} - A\frac{mc^2}{2kT^2}\frac{\partial T}{\partial \mathbf{r}}\right) + \left(\frac{\mathbf{F}}{m} - \left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0\right)\cdot\mathbf{c}\frac{mA}{kT} - \frac{mA}{kT}\mathbf{cc} : \frac{\partial\,\mathbf{v}_0}{\partial \mathbf{r}}\right] = 0 \quad (6)$$

This implies that all the coefficients of successive powers of the peculiar velocity must identically vanish; we therefore obtain the following four equations:

$$\frac{mA}{kT}\mathbf{cc} : \frac{\partial \mathbf{v}_0}{\partial \mathbf{r}} = 0, \quad (7a)$$

$$\mathbf{v}_0\cdot\frac{\partial A}{\partial \mathbf{r}} = 0 \;\Rightarrow\; \mathbf{v}_0 \perp \frac{\partial A}{\partial \mathbf{r}}, \quad (7b)$$

$$A\frac{mc^2}{2kT^2}\mathbf{c}\cdot\frac{\partial T}{\partial \mathbf{r}} = 0 \;\Rightarrow\; \frac{\partial T}{\partial \mathbf{r}} = 0, \quad (7c)$$

$$\frac{\partial A}{\partial \mathbf{r}} + \frac{A}{kT}\left(\mathbf{F} - m\left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0\right) = 0. \quad (7d)$$

These equations are as general as equation (6) but are indeed extremely useful; in particular equation (7c) states, as logical, that at equilibrium the temperature must be constant and equation (7d) establishes an equilibrium relation between the various external and internal forces acting over the gas. Moreover it provides a relation between the chemical potential and all the other forces.

We consider now an electron gas, in condition of cylindrical symmetry ($\hat{\mathbf{z}}$ being the symmetry axis), described through cylindrical coordinates $\mathbf{r} = [r, \vartheta, z]$. The external force is given by a constant and uniform magnetic field along the symmetry axis, $\mathbf{B} = B\hat{\mathbf{z}}$; excluding correlation and exchange effects, the remaining internal force is given by the self-consistent Vlasov electric field.

Under these conditions all the quantities in equation (7d) will depend at most on r alone. This allows a simple form of the average velocity to be found through equation (7a); expanding the double dot product, and taking into account that to have stable equilibrium no radial velocity must be present (this practically corresponds to study systems whose quantum number l is 0), we get:

see equation (8) below

$$\mathbf{cc} : \frac{\partial \mathbf{v}_0}{\partial \mathbf{r}} = c_r c_r \frac{\partial v_{0r}}{\partial r} + c_\vartheta c_\vartheta\left(\frac{1}{r}\frac{\partial v_{0\vartheta}}{\partial \vartheta} + \frac{v_{0r}}{r}\right) + c_z c_z\frac{\partial v_{0z}}{\partial z} + c_r c_\vartheta\left(r\frac{\partial}{\partial r}\frac{v_{0\vartheta}}{r} + \frac{1}{r}\frac{\partial v_{0r}}{\partial \vartheta}\right) + c_\vartheta c_z\left(\frac{1}{r}\frac{\partial v_{0z}}{\partial \vartheta} + \frac{\partial v_{0\vartheta}}{\partial z}\right) + c_r c_z\left(\frac{\partial v_{0z}}{\partial r} + \frac{\partial v_{0r}}{\partial z}\right) = c_r c_\vartheta\left(r\frac{\partial}{\partial r}\frac{v_{0\vartheta}}{r}\right) + c_r c_z\frac{\partial v_{0z}}{\partial z} = 0 \quad (8)$$

so that we have:

$$\frac{\partial v_{0z}}{\partial z} = 0 \;\Rightarrow\; v_{0z} = \text{constant}$$

$$\frac{\partial v_{0\vartheta}}{\partial r} - \frac{v_{0\vartheta}}{r} = 0 \;\Rightarrow\; v_{0\vartheta} = \boldsymbol{\omega} \times \mathbf{r}$$

where $\boldsymbol{\omega} = \omega\hat{\mathbf{z}}$ is a constant vector, directed along the symmetry axis, representing an angular velocity for the gas. In total the average velocity can be written as:

$$\mathbf{v}_0 = v_{0z}\hat{\mathbf{z}} + \boldsymbol{\omega} \times \mathbf{r}. \tag{9}$$

Now, inserting this expression and the Lorentz force (external magnetic field + unknown internal self-consistent electric field) into equation (7d) we are left only with the following scalar equilibrium equation that connects the gradient of the chemical potential, the Lorentz force and the centrifugal force:

$$kT\frac{\mathrm{d}}{\mathrm{d}r}(\ln A) - e(E_r + \omega r B) + m_\mathrm{e}\omega^2 r = 0. \tag{10}$$

Expressing the electric field as the gradient of a potential V

$$E_r = -\frac{\mathrm{d}V}{\mathrm{d}r}, \tag{11}$$

for which we assume the following condition

$$\lim_{r\to 0^+} V(r) = 0, \tag{12}$$

equation (10) can be solved to give

$$\mu(r,T) = \mu(0,T) + eV(r) + \frac{1}{2}m\omega(\omega - \omega_\mathrm{c})r^2 \tag{13}$$

where $\mu(0,T)$ is the chemical potential for an uniform electron gas, *i.e.* the value of the chemical potential when no forces are present, and $\omega_\mathrm{c} = eB/m_\mathrm{e}$ is the cyclotron frequency, representative of the external magnetic field. Inserting equation (13) into the Fermi distribution function and integrating all over the space of velocities (making use of first order Sommerfeld's lemma and assuming the usual boundary conditions in momentum space for the distribution function) the electron density can be found:

$$n(r) = \frac{8\pi}{3h^3}\left(2m_\mathrm{e}\left(eV(r) + \frac{1}{2}m_\mathrm{e}\omega(\omega-\omega_\mathrm{c})r^2 + \mu_0\right)\right)^{3/2} \times \left(1 + \frac{(\pi kT)^2}{8}\left(eV(r) + \frac{1}{2}m_\mathrm{e}\omega(\omega-\omega_\mathrm{c})r^2 + \mu_0\right)^{-2}\right) \tag{14}$$

here $\mu_0 = \mu(0,0) \equiv E_\mathrm{F}$ is the Fermi energy. In this way we have determined through kinetic theory methods the extension to non-zero temperature cases of the Thomas-Fermi equation as generalized by Marshak and Bethe [9] and as studied by Feynman, Metropolis and Teller [10]; moreover we added the important case of a non zero-average velocity and the effects of a magnetic field.

As example of this approach we consider here the case of a completely degenerate electron gas. Assuming a complete degeneracy, we can put $T = 0$ in equation (14) to obtain

$$eV(r) = n(r)^{2/3}\frac{1}{2m_\mathrm{e}}\left(\frac{3h^3}{8\pi}\right)^{2/3} - \frac{1}{2}m_\mathrm{e}\omega(\omega-\omega_\mathrm{c})r^2 - \mu_0. \tag{15}$$

To get a self-consistent description of the gas we now make use, in a system together with the preceding equation, of Poisson's equation written in cylindrical coordinates:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right)V(r) = \frac{e}{\varepsilon_0}n(r); \tag{16}$$

solving the system we have finally an equation for the electron density:

$$\frac{\mathrm{d}^2 n}{\mathrm{d}r^2} - \frac{1}{3n}\left(\frac{\mathrm{d}n}{\mathrm{d}r}\right)^2 + \frac{1}{r}\frac{\mathrm{d}n}{\mathrm{d}r} = 3m_\mathrm{e}\left(\frac{8\pi}{3h^3}\right)^{2/3} \times n^{1/3}\left(2m_\mathrm{e}\omega(\omega-\omega_\mathrm{c}) + \frac{e^2}{\varepsilon_0}n\right) \tag{17}$$

which must be coupled with the following conditions for n:

1. $\lim_{r\to 0^+} n(r) = n_0$ to fix the total number of electrons (18a)
2. $\lim_{r\to 0^+} \frac{\mathrm{d}n}{\mathrm{d}r} = 0$ to warrant the symmetry (18b)
3. $\lim_{r\to +\infty} n(r) = 0$ to warrant confinement (18c)

we will see in the following paragraph that only the first two conditions are really necessary to solve equation (17) in those regions of the (angular velocity-cyclotron frequency) space where confinement is assured.

3 Analytical results

Even if equation (17) can be solved exactly only numerically, we can nonetheless find some interesting analytical results.

First of all let us investigate the concavity of the density near the origin; only two cases are obviously possible:

1. $n''(0) > 0$. In this case the first derivative of the density must be positive near the origin, so that the lhs of equation (17) is positive in the neighbourhood of the origin; hence even the rhs must be the like. Then, because the density is positive-defined and must vanish at infinity, it must reach a maximum and then decrease. However in a point of maximum n' must vanish and n'' must be negative; this would imply that in a neighbourhood of the maximum point the lhs of equation (17) is negative but this is impossible because, until $n(r) > n(0) > 0$ (condition to have a maximum) the rhs remains positive.

2. $n''(0) < 0$. In this case the first derivative of the density is negative near the origin and so negative near the origin is the lhs of equation (17) too. The same must be of the rhs. Because the density must vanish at infinity and because it cannot have local minima it must always be decreasing, as logic suggests. In fact in a minimum point the first derivative is zero and the second derivative is positive, so that near the minimum point the lhs of equation (17) is positive and positive must also be the rhs, but this is impossible since near the minimum point $0 < n(r) < n(0)$ and the rhs would remain negative.

In view of the above considerations we see that only case 2 is compatible with the boundary conditions and that the density cannot have minima or maxima and that it can only decrease. Case 2 is expressed analytically by:

$$n''(0) < 0 \;\Rightarrow\; \lim_{r\to 0^+} 3m_{\mathrm{e}} \left(\frac{8\pi}{3h^3}\right)^{2/3} \times n^{1/3}\left(2m_{\mathrm{e}}\omega(\omega-\omega_{\mathrm{c}}) + \frac{e^2}{\varepsilon_0}n\right) < 0 \quad (19)$$

or by

$$2m_{\mathrm{e}}\omega(\omega-\omega_{\mathrm{c}}) + \frac{e^2}{\varepsilon_0}n_0 = 2m_{\mathrm{e}}\omega(\omega-\omega_{\mathrm{c}}) + m_{\mathrm{e}}\omega_{\mathrm{p}}^2 < 0 \quad (20)$$

where

$$\omega_{\mathrm{p}} = \sqrt{\frac{n_0 e^2}{m_{\mathrm{e}}\varepsilon_0}}$$

is the plasma frequency at the density $n_0 = n(0)$.

Inequality (20) gives the domain of values of the angular velocity and of the cyclotron frequency to have confinement of the electron gas, to realize, in other words, a condition in which the magnetic field is sufficient to counterbalance the effects of the centrifugal force and of the self-consistent repulsive electrical potential.

Inequality (20) is satisfied by

$$\omega_2 \le \omega \le \omega_1 \quad (21)$$

with

$$\omega_{1,2} = \frac{\omega_{\mathrm{c}}}{2}\left(1 \pm \sqrt{1 - 2\left(\frac{\omega_{\mathrm{p}}}{\omega_{\mathrm{c}}}\right)^2}\right). \quad (22)$$

Figure 1 shows an universal confinement region given by the two curves (22) [4,5].

Figure 2 shows the confinement region for unity of density at the origin, as a function of the confining magnetic field.

As one could expect, the values permitted to the angular velocity ω increase with the confining magnetic field and *vice versa*. It can also be verified that the higher $n(0)$ the higher must be the confining field.

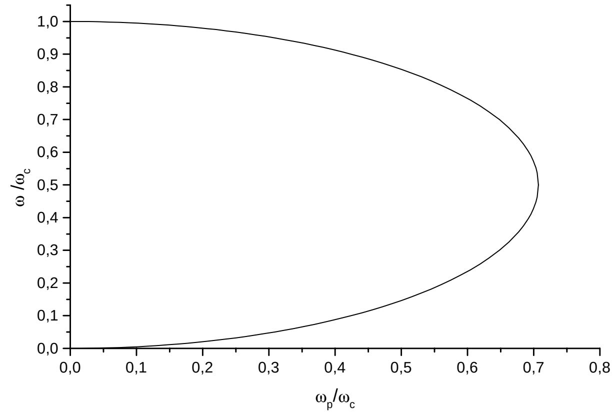

Fig. 1. Universal confinement region.

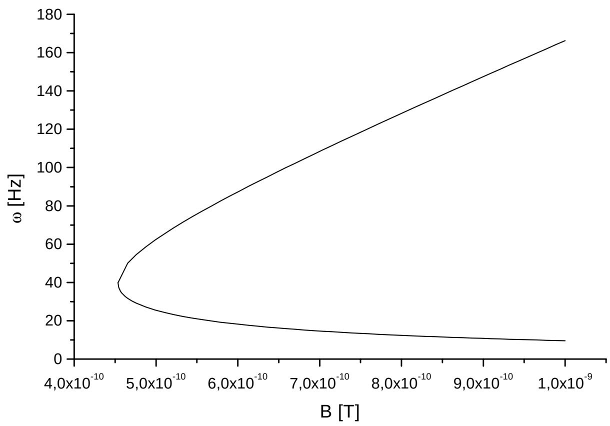

Fig. 2. Confinement region for $n(0) = 1\ \mathrm{m}^{-3}$.

Now, having established and studied the domain of confinement, we want to find an approximate analytical solution to equation (17).

We look for solutions in the small r range; here, since $n(0)$ is limited and $n'(0) = 0$, we can neglect the term $\propto (n')^2/n$ in equation (17); moreover we notice that the rhs of equation (17) is a non-linear function of n; in the neighbourhood of the origin we can expand this non-linear function in a Taylor series around n_0 and truncate it at order 1; we remember that:

$$n^{1/3}(a + bn) \cong \frac{1}{3}(2a - b + (a + 2b)n).$$

In this way equation (17) can be written as:

$$n'' + \frac{n'}{r} = \frac{c}{3}(2a - b + (a + 2b)n)$$

where $a = 2m_{\mathrm{e}}\omega(\omega-\omega_{\mathrm{c}})$, $b = e^2/\varepsilon_0$ and $c = 3m_{\mathrm{e}}(8\pi/3h^3)^{2/3}$.

This simplified equation, with the initial conditions stated above, has as solution the following (tilde means the solution is approximate):

$$\tilde{n}(r) = \frac{1}{4b + a}\left(b - 2a + 3(a + b)J_0\left(r\sqrt{-\frac{c}{3}(4b + a)}\right)\right) \quad (23)$$

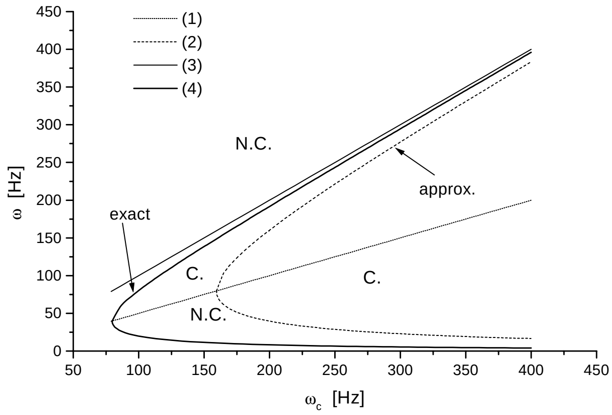

Fig. 3. Confinement regions for $n(0) = 1\ \mathrm{m}^{-3}$: C. = confinement; N.C. = no confinement. Lines: (1) $\omega = 0.5\omega_c$, (2) confinement region boundary for the approximate equation, (3) plane bisector, (4) confinement region boundary for the exact equation.

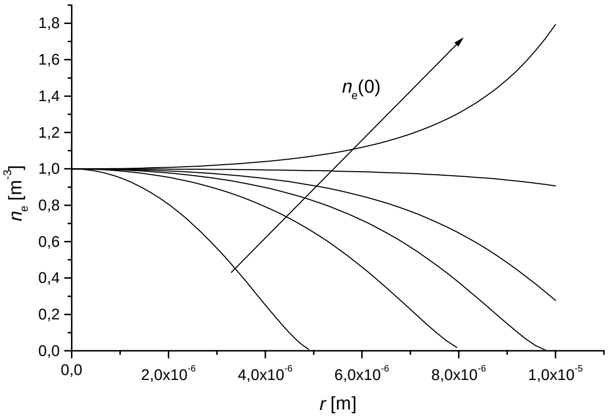

Fig. 4. Density profiles: transition between confinement and non-confinement state for various values of the density at the origin (10, 18, 20, 21, 22 and 23 m^{-3} respectively) and for $\omega_c = 375$ Hz, $\omega = 0.5 \times 375$ Hz.

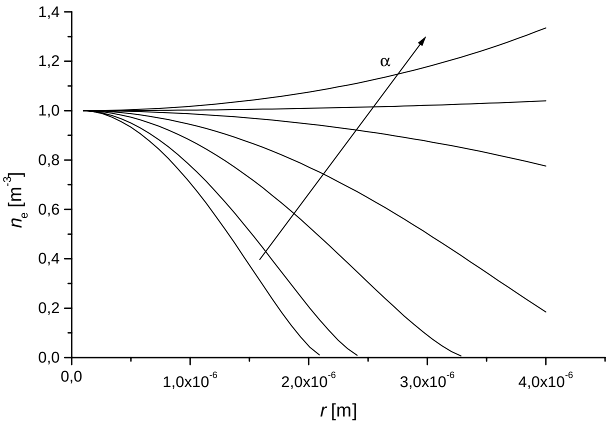

Fig. 5. Density profiles: transition between confinement and non-confinement state for various values of the angular velocity $\omega = \alpha\omega_c$ (α equals to 0.7, 0.8, 0.9, 0.95, 0.98, 0.99 and 1 respectively) and for $\omega_c = 375$ Hz and $n(0) = 1\ \mathrm{m}^{-3}$.

where J_0 is the zeroth-order Bessel function of the first kind.

We see that the solution has in itself the modified domain for confinement for the simplified equation; in fact we remember that [11]:

1. $J_0(\mathrm{i}x) \in R \quad \forall x \geq 0,$
2. $J_0(\mathrm{i}x) \equiv I_0(x) \geq 1 \quad \forall x \geq 0,$
3. $J_0(x) \leq 1 \quad \forall x \geq 0.$

where $I_0(x)$ is the zeroth-order Bessel function of the second kind. From equation (23) we see therefore that to have a meaningful approximate solution we must have:

$$4b + a < 0$$

which gives us the restricted domain of confinement for the approximate equation and solution as shown in Figure 3.

As it is good to find, the confinement region for the approximate equation lies inside the larger region of the exact equation. Moreover we see that the upper bound of the permitted angular velocities has the bisector of the plane as asympt.

Obviously the validity of the approximate solution must be restricted to zones far away from the point of its first zero. We will see in the next paragraph that the approximate analytical solution is quite good in comparison to the numerical solution and that it is mathematically good also outside the confinement region, even if the physical meaning is lost.

4 Numerical results

Equation (17) was solved also numerically by use of standard methods and packages such as 5th order Runge-Kutta and Livermore stiff ODE solvers [12]. This numerical solution was then used to estimate the error made by the numerical techniques by means of the evaluation of its residual; we fitted the numerical solution to a 5th order polynomial and tested its accuracy against the numerical solution; then we inserted it into the initial equation and found that the residual error made herein didn't exceed the value of $2 \times 10^{-26}\ \mathrm{m}^{-3}$ with respect to values of the order of $1\ \mathrm{m}^{-3}$.

Figure 4 shows the behaviour of the solution for various values of one parameter.

It shows how the solution converges or diverges, giving confirmation of confinement or non-confinement respectively (this last case loosing its physical meaning, but retaining the mathematical one, as happens for the approximate analytical solution), for fixed values of the angular velocity and of the confining field, but for different values of the density of electrons at the origin.

Figure 5 shows how the solution converges or diverges, giving confirmation of confinement or non-confinement respectively (this last case loosing its physical meaning, but retaining the mathematical one, as happens for the

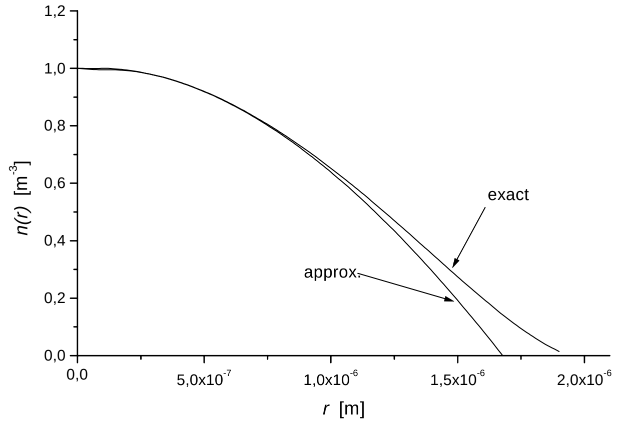

Fig. 6. Density profiles: comparison between approximate analytical solution and the equivalent numerical one.

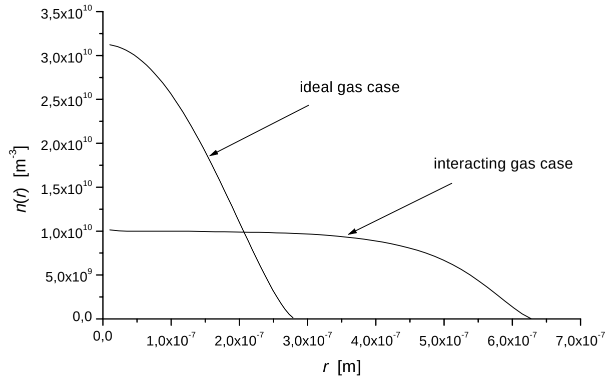

Fig. 7. Comparison between density profiles for the ideal electron gas and the non-ideal one.

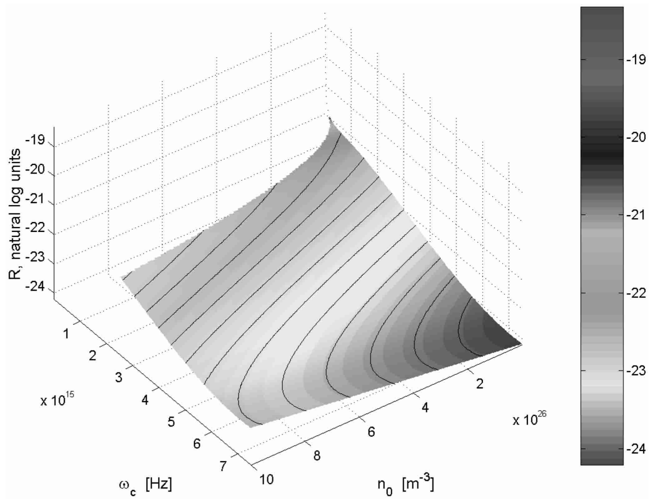

Fig. 8. Natural logarithm of the confinement radius as a function of cyclotron frequency and density n_0; black curves are iso-radius lines.

approximate analytical solution), for fixed values of the confining field and of the density at the origin, but for different values of the angular velocity.

Figure 6 compares the approximate small r-range analytical solution and the numerical one for $n(0) = 1$ m^{-3} and for $2\omega = \omega_c = 375$ Hz.

As can be seen there is good agreement between the two in a large part of the domain.

5 The case of the ideal Fermi gas

We want here to examine also the interesting case of the ideal electron gas; this task is easily accomplished by setting V and T to zero in equation (14). We get:

$$n(r) = \frac{8\pi}{3h^3}\left(2m_e\left(\mu_0 + \frac{1}{2}m_e\omega(\omega-\omega_c)r^2\right)\right)^{3/2} \quad (24)$$

where obviously $0 < r < R$,

$$R = \sqrt{\frac{2\mu_0}{m_e\omega(\omega_c - \omega)}},$$

and $\omega_c > \omega$ to have confinement.

We see that the Pauli Principle, taken into account by the Fermi statistics, is shown through an effective repulsion between the electrons that prevents them from collapsing together at $r = 0$; this is a confirmation of the fact that the ideal Fermi gas then is not an ideal gas in the usual sense, but has strong internal features that distinguish it from the ideal non-degenerate gas of identical particles [13].

In Figure 7 the density profiles of the ideal and non-ideal electron gas are compared for equal total number of particles per unit length $N \cong 0.0023$ m^{-1} and for equal magnetic field and angular velocity ($\omega = 4.2 \times 10^6$ Hz, $\omega_c = 8 \times 10^6$ Hz). The non-ideal gas profile has been numerically obtained with $n(0) = 10^{10}$ m^{-3}; this profile has been then integrated to evaluate N so that, from this value, the chemical potential for the ideal gas could have been calculated.

As results, the non-ideal gas profile is lower but larger while the ideal gas one is narrower and higher as one could expect. We report also that these differences between the two profiles increase with N, as logical.

6 Conclusions

In this paper we have presented some numerical results of a method developed from semiclassical kinetic-theory to study certain equilibrium properties of a non-ideal degenerate fermion gas. In particular we have focused our attention to the influence of angular velocity and external magnetic field upon the radial confinement of the gas itself. The total number of particles, the intensity of the trapping magnetic field and the value of the average angular velocity all influence heavily the confinement radius. As a whole we have studied the variations of this quantity as a function of ω, ω_c and n_0.

Further numerical investigations permit us to propose the approximate Figure 8 which gives the natural logarithm of the confinement radius as a function of n_0 and ω_c for the fixed ratio of $\omega/\omega_c = 1/2$; the curves above the surface are iso-radius lines. This approximate figure is the result of a best fitting of hundreds of $[n_0, \omega_c, \ln(R)]$ points taken from the confinement region with the further restriction that $\omega/\omega_c = 1/2$. The density n_0 was chosen in the range $[10^{18}, 10^{27}]$ m^{-3} and the cyclotron frequency in the range $[10^{10}, 10^{16}]$ Hz.

We have also shown the differences in the profiles from the ideal-gas case.

For the strong differences from the classical gas case see [5].

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